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Vacuum polarization and the absence of free quarks

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This paper is addressed to the question of why isolated quark partons are not seen. It is argued that in vector gauge theories it is possible to have the short-distance and light-cone behavior of quark fields without real quark production in deep-inelastic reactions. The physical mechanism involved is the flow of vacuum-polarization currents which neutralize any outgoing quarks. Our ideas are inspired by arguments due to Schwinger and an intuitive picture of Bjorken. Two-dimensional (1 space, 1 time) vector gauge field theories provide exactly soluble examples of this phenomenon. The resulting picture of deep-inelastic final states predicts jets of hadrons and logarithmically rising multiplicities as conjectured by Bjorken and Feynman.

I. INTRODUCTION

The hypothesis that partons carry quark quantum numbers[1] poses a deep puzzle. An optimist can foresee a situation in the near future where deep-inelastic structure functions indicate that hadrons consist of pointlike quarks even though no isolated quarks are detected experimentally. The parton explanation of Bjorken scaling rests on the idea that partons experience just soft, finite forces.[2] This suggests that when a parton absorbs a very virtual photon of momentum Q it should propagate essentially freely for distances which grow linearly with Q. One would naively expect that once this distance exceeds the size of a hadron, then quarks could be produced. If, in fact, quarks are not produced one should ask whether the measurement of quark quantum numbers in deep-inelastic processes and the absence of free quarks is consistent with quantum field theory. It is the purpose of this article to argue that this consistency is indeed possible and can be demonstrated in a solvable field theory.

The field-theoretic mechanism which allows this consistency was discovered by Schwinger.[3] Schwinger observed that the quantum vacuum of a gauge field theory may be so polarizable that charge can be completely screened. The vector mesons of the gauge theory then acquire a mass. We shall argue that this mechanism can also remove the underlying quarks from the physical spectrum of states. This mechanism involves the large-distance properties of the field theory and does not modify the light-cone and short-distance (i.e., scaling) properties of the theory. In our first example the quark quantum numbers will be replaced by a single charge (later to be replaced by charm). In this model the analog of triality will be fermion number. We present a second example containing a triplet of quarks and show that the physical states are mesons and baryons ($q\bar{q}$ and qqq states).

The physical picture of Schwinger's mechanism is best illustrated for the annihilation reaction $e^+e^- \to$ hadrons at center-of-mass energy Q. The virtual photon decays into a quark-antiquark pair which move apart at almost the velocity of light. An "electric" field[4] develops between them and begins producing $q\bar{q}$ pairs out of the vacuum. As the original $q\bar{q}$ separate, a line of polarized pairs forms between them. The polarization charge eventually catches up with the outgoing quarks and combines with them to form hadrons. The time necessary for the original quarks to be neutralized in this way increases linearly with Q. This softness of the final-state mechanism insures that the matrix elements of deep-inelastic scattering and annihilation behave *as if* the original quarks were produced. The $q\bar{q}$ pairs making up the polarization current have low relative subenergies and bind to form mesons.

This paper is organized as follows. In Sec. II we illustrate the problems involved in avoiding quark production in a conventional multiperipheral parton model. A parton model due to Bjorken[5] in which partons interact through vector exchanges is then discussed. In Sec. III two-dimensional (1 space, 1 time) quantum electrodynamics is formulated as a model of the strong interactions of quarks and its solution in terms of free fields is presented. Then a semiclassical treatment of $e^+e^- \to$ bosons is presented and we demonstrate how the outgoing quarks are neutralized. Some properties of the distribution of the produced bosons in momentum space are obtained. In particular, they populate the available rapidity axis with a constant, nonzero density. In Sec. IV properties of free fermi fields in two dimensions are discussed. The short-distance and light-cone behavior of various matrix elements are obtained for later comparisons with similar calculations in the interacting theory. In Sec. V we compute Green's functions and deep-inelastic matrix elements in two-dimensional quantum electrodynamics. We demonstrate that although the short-distance and light-cone behavior of multiple-current matrix elements are characteristic of free Fermi fields, the intermediate states of these matrix elements involve only massive bosons. This proves our claim that the fermions are absent from the physical spectrum of the theory. We also calculate the deep-inelastic structure function for massive bosons and find that it tends to a constant as $\eta = -q^2/2q\cdot p \to 0$. The spectrum of bosons in $e^+e^- \to$ boson + anything is computed and is in agreement with the semiclassical calculation of Sec. III. In Sec. VI we present a three-triplet model in which the physical sector consists of bosons $(q\bar{q})$ and fermions (qqq). In Sec. VII we discuss a semiclassical model for the polarization currents in four dimensions. Although the semiclassical model cannot describe short-distance behavior, it shows that the long-distance mechanisms which neutralize quarks may be possible in four dimensions. Section VIII contains concluding remarks. Here we discuss the problems involved in generalizing Schwinger's mechanism to a mechanism for screening triality in four dimensions. In Appendix A the solution of two-dimensional quantum electrodynamics in the Coulomb gauge is given.

II. CONVENTIONAL MODEL AND BJORKEN'S SUGGESTION

We first illustrate the problems involved in preventing the presence of isolated quarks in the final states of deep-inelastic reactions by considering a model based on multiperipheral dynamics.[6] In this model partons only interact if they have small relative subenergies. Consider the $q\bar{q}$ pair initially produced by the decay of a very virtual timelike photon of mass $\sqrt{Q^2}$. Each parton subsequently evolves separately through low-subenergy processes. The outgoing fast quark generates another of lower momentum which produces a third of yet lower momentum, etc. (Fig. 1). At each stage the emitted parton carries a finite fraction of the longitudinal momentum of its predecessor (Fig. 1). It was hoped that each cascade would eventually produce "wee" partons which could form a bridge and neutralize the quark charges. The difficulty with this suggestion can be seen by estimating the time until the first event in each cascade. Simple time-dilation arguments say that the time necessary for the first cascade grows linearly with ηQ, where η is the longitudinal fraction of the first cascade product (Fig. 2). Thus, when Q becomes sufficiently large, the outgoing partons are so far apart by the time the cascade initiates that there is no chance for the two cascades to overlap in coordinate space. Hence, this traditional multiperipheral cascade which develops from the fast quarks toward the central region ("outside-inside" cascade, Fig. 1) fails.

One possible mechanism which avoids this dilemma has been proposed by Bjorden.[5] It involves the presence of vector forces between partons. Vector exchanges allow partons of arbitrarily high subenergy to interact with finite probability. In particular, the amplitude for the outgoing quark to produce a vector meson of longitudinal fraction η

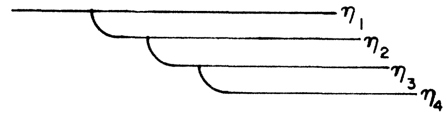

FIG. 2. Multiperipheral cascade. η_i label the longitudinal fractions of the partons.

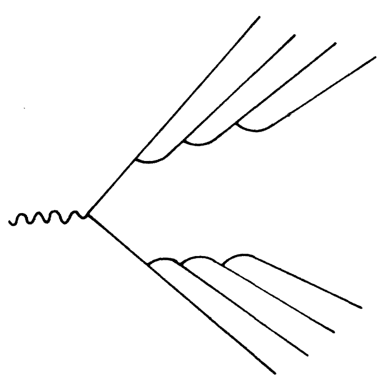

FIG. 1. Multiperipheral parton cascade in $e^+e^- \to$ anything. The wee partons in different cascades do not overlap in configuration space and cannot interact.

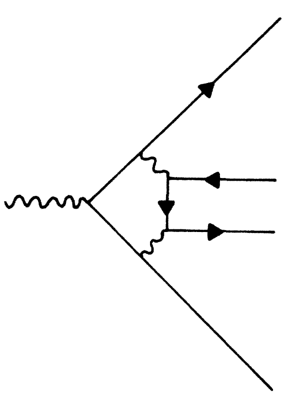

FIG. 3. Vector exchanges create a pair between the original partons.

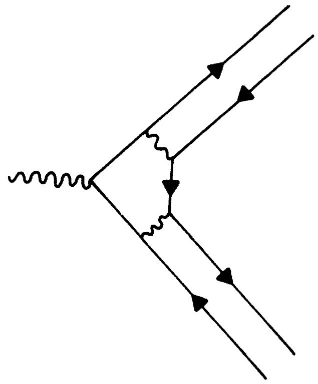

FIG. 4. Figure 3 with two pairs at small subenergy. The probability for this process falls to zero by several powers of Q^2 faster than Q^{-2}.

remains finite as η tends to zero. Therefore, the time needed for the original quark to emit a wee ($\eta < Q^{-1}$) vector meson is of order unity ($\eta Q \sim Q^{-1} Q \sim 1$). Suppose now that each outgoing parton radiates a wee vector meson in a time of order unity. The vector fields can then create a parton-antiparton pair as in Fig. 3. The probability for this event is large in the sense that the contribution this graph makes to the total cross section $\sigma_{e^+e^-}$ scales as Q^{-2}. The subenergy of the newly created pair is small. The pair has uniform probability to have any rapidity between the rapidities of the initial quarks. Since the subenergy of the new pair is small, it is almost impossible to combine the members of the new pair with the initial quarks to form two outgoing mesons (Fig. 4). At this stage the rapidity axis has two large vacant sections. Therefore, one expects the vector forces to create additional pairs in these regions of phase space (Fig. 5). These processes can continue to occur until the available section of the rapidity axis is uniformly populated with pairs. Once this occurs the quarks can form a set of outgoing mesons as shown in Fig. 6.

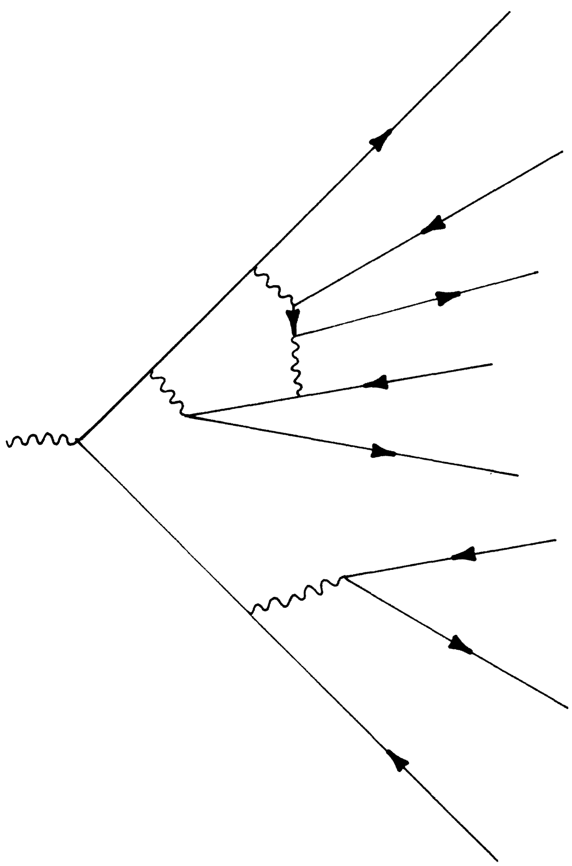

FIG. 5. Second-generation "cascade" in vector-exchange model.

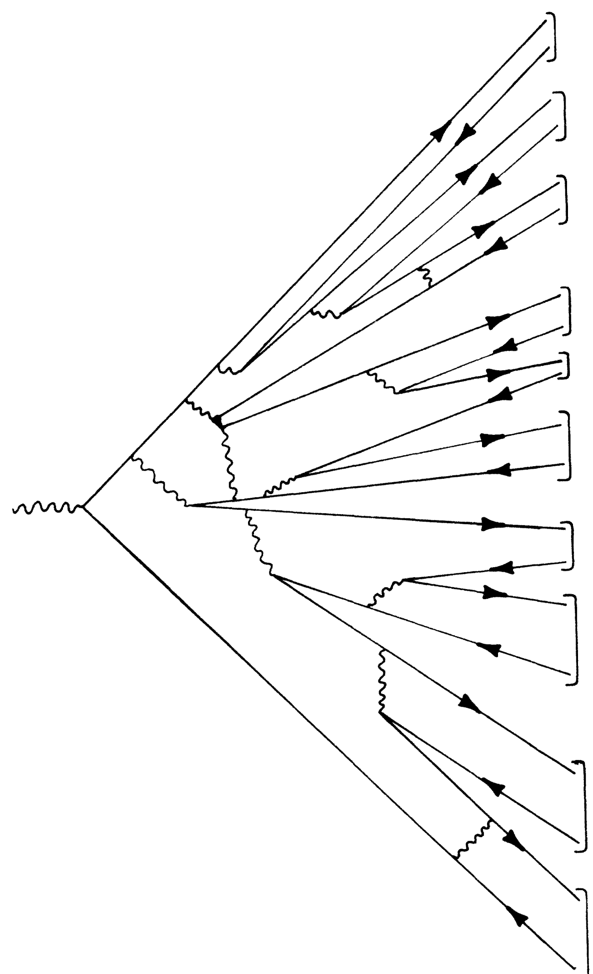

FIG. 6. "Inside-outside" vector-exchange cascade after time duration $\approx Q$. The indicated pairs are at finite subenergies and can form mesons which uniformly populate the available rapidity axis.

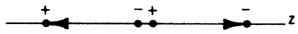

FIG. 7. Configuration-space visualization of Fig. 3.

Now consider the space-time development of the final state. Initially the original partons are receding from one another at almost the speed of light. After separating a certain finite average distance, the first generation pair is produced between them (Fig. 7). It is convenient to think of this pair as a dipole $g(r_+ - r_-)$ where $r_\pm$ refers to the position of the quark (antiquark) and g is the quark–gauge-field coupling constant. Subsequently, more pairs are produced and the region containing polarized pairs between the receding original partons spreads out along a line between them (Fig. 8). This type of process can be thought of as an "inside-outside" cascade. It will be convenient to define a dipole density on this line $\phi(z, t)$. The polarization charge density and current are then,

$$\rho = \frac{\partial \phi}{\partial z}, \quad j = -\frac{\partial \phi}{\partial t}. \tag{1}$$

The outer ends of the line of polarized pairs carry net polarization charge which eventually catches up to the original receding charges and neutralizes them (Fig.9). The process we have described must not only be probable, but it must occur with probability one. We will argue that this is indeed the case when the Schwinger phenomenon[3] occurs.

III. A PHYSICAL PICTURE OF TWO-DIMENSIONAL QUANTUM ELECTRODYNAMICS

In this section we will consider two-dimensional massless quantum electrodynamics[3] as a model of the Schwinger mechanism. In this case, unlike four-dimensional quantum electrodynamics, the Schwinger phenomenon occurs for all values of the coupling constant.

Two-dimensional massless quantum electrodynamics is described by a two-component Fermi field ψ and a vector potential A_μ. The familiar Lagrangian reads

$$\mathcal{L} = \bar{\psi}\, i\gamma^\mu \partial_\mu \psi - \tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} - g\bar{\psi}\gamma^\mu \psi A_\mu , \tag{2}$$

and the equations of motion are

$$\begin{aligned} &\gamma^\mu (i\partial_\mu - gA_\mu)\psi = 0 , \\ &j^\mu \equiv g\bar{\psi}\gamma^\mu \psi = \partial_\nu F^{\nu\mu} , \\ &F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu . \end{aligned} \tag{3}$$

FIG. 8. Subsequent development of a line of dipoles between the original parton-antiparton pair.

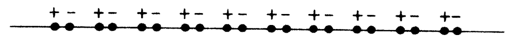

FIG. 9. After time duration $\approx Q$ the line of dipoles catches the outgoing pair. This is the configuration-space visualization of Fig. 6.

The definition of the electric current requires some care in order to satisfy gauge invariance. Define the point-separated current[3]

$$j^\mu(x) = \operatorname{sym} \lim_{\epsilon \to 0} g\bar{\psi}(x+\epsilon)\gamma^\mu \times \exp\left(ig \int_x^{x+\epsilon} A^\mu dx_\mu \right) \psi(x) , \tag{4}$$

where ϵ is a spacelike vector. The equation of motion for j^μ follows[3]:

$$(\Box + m^2) j^\mu(x) = 0 , \tag{5}$$

where $m^2 = g^2/\pi$. Thus, the spectrum of the theory contains free massive bosons. If an external current $j^\mu_{\rm ext}(x)$ is introduced into the theory, the equation of motion for j^μ becomes

$$(\Box + m^2) j^\mu(x) = -m^2 j^\mu_{\rm ext}(x) . \tag{6}$$

We will introduce a dipole density $\phi(x)$ in analogy with Eq. (1),

$$\begin{aligned} &j^\mu = \epsilon^{\mu\nu}\partial_\nu \phi , \quad j^\mu_{\rm ext} = \epsilon^{\mu\nu}\partial_\nu \phi_{\rm ext} , \\ &\phi(z, t) = \int_{-\infty}^{z} j^0(z', t) dz' . \end{aligned} \tag{7}$$

It follows that ϕ satisfies the equation

$$(\Box + m^2)\phi = -m^2 \phi_{\rm ext} , \tag{8}$$

and ϕ is proportional to a canonical massive boson field. ϕ satisfies commutation relations

$$[\phi(z, t), \dot{\phi}(z', t)] = i m^2 \delta(z - z'). \tag{9}$$

The equation of motion for A_μ in the Lorentz gauge follows from Eq. (3),

$$\Box A = j + j_{\rm ext} . \tag{10}$$

Therefore, from Eqs. (5) and (10) we see that $m^2 A + j$ satisfies a massless Klein-Gordon equation. Thus, if we define $\tilde{\phi}$ by the equation $m^2 A_\mu + j_\mu = \epsilon_{\mu\nu}\partial^\nu \tilde{\phi}$, then

$$\Box \tilde{\phi} = 0. \tag{11}$$

Furthermore, since[3]

$$[j_0(z, t), j_1(z', t)] = -i m^2 \delta'(z - z') \tag{12}$$

and

$$[A_0(z, t), A_1(z', t)] = 0, \tag{13}$$

it follows that

$$[\tilde{\phi}(z, t), d\tilde{\phi}(z', t)/dt] = -i m^2 \delta(z - z'). \tag{14}$$

The solution for the Fermi field has a particularly simple form which illustrates the utility of the fields ϕ and $\tilde{\phi}$. It is easy to see that Eq. (3) can be solved in terms of a free canonical Fermi field χ given by

$$\chi(x)=\exp[ig\alpha\Phi(x)]\psi(x), \tag{15}$$

where $\alpha=\gamma_0\gamma_1$ and $\Phi=\pi(\tilde{\phi}-\phi)/g^2$. This result will prove useful in later discussions of matrix elements of bilinears such as $\bar{\psi}\psi$ and $\bar{\psi}\gamma^\mu\psi$.

To see how polarization currents in the model prohibit individual quark production, consider the example $e^+e^-\to$ anything. We begin with a very energetic state of a quark and an antiquark emanating from one space-time point. The outgoing initial quarks will be replaced by a c-number external current consisting of two point charges which travel at (almost) the speed of light in opposite directions. Then,

$$\begin{aligned} j_0^{\text{ext}}&=g\delta(z-t),\quad j_1^{\text{ext}}=g\delta(z-t)\qquad \text{for } z>0,\\ j_0^{\text{ext}}&=-g\delta(z+t),\quad j_1^{\text{ext}}=g\delta(z+t)\qquad \text{for } z<0, \end{aligned} \tag{16}$$

The external dipole density is then

$$\phi_{\text{ext}}=-g\theta(t+z)\theta(t-z) \tag{17}$$

so the induced q-number dipole density satisfies

$$(\Box+m^2)\phi=gm^2\theta(t+z)\theta(t-z). \tag{18}$$

The radiation field resulting from Eq. (18) is described by a coherent state. The amplitude of the coherent state satisfies the same equation as ϕ itself with vacuum boundary conditions for negative times. The solution to Eq. (18) is

$$\phi(x)=2g\int\left(\frac{1}{p^2-m^2}-\frac{1}{p^2}\right)e^{-ip\cdot x}\frac{d^2p}{(2\pi)^2}, \tag{19}$$

where the boundary conditions are $\phi(x)=0$ unless $t>0$ and $t^2>z^2$. In coordinate space

$$\phi(x)=g\theta(t+z)\theta(t-z)-g\Delta_R(m^2,x^2), \tag{20}$$

where Δ_R is the retarded commutator. Δ_R is well approximated by $\theta(t+z)\theta(t-z)$ near the light cone and by $e^{-m|x|}$ for $|x|$ large. Thus, the resulting amplitude vanishes near the light cone and tends to a constant when $|x|>m^{-1}$. Since ϕ is also a Lorentz-invariant function, the lines of constant ϕ are the hyperbolas $t^2-z^2=$constant.

We can now understand the time development of the polarization charge. At small time $t<m^{-1}$, the dipole density ϕ as a function of z is small. For $t\approx m^{-1}$ the dipole density midway between the receding pair is $\approx g$ as in Fig. 10. As t grows the dipole density becomes constant over the entire spatial line between the pair (Fig. 10). The polarization charge $\partial_1\phi$ follows the outgoing initial

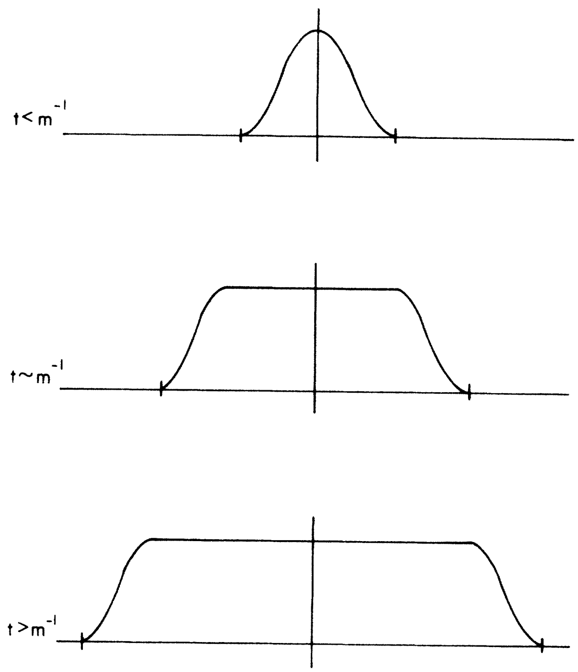

FIG. 10. The spreading of the dipole density ϕ with time. The polarization charge is nonzero in regions of space where ϕ varies.

fermions. The region of variation of ϕ which contains the polarization charge is confined to an interval of order $(tm^2)^{-1}$ from the outgoing partons. This follows from the fact that ϕ is constant on hyperbolas which approach the light cone asymptotically. The magnitude of each polarization charge is given by the difference of ϕ across these intervals. Since ϕ vanishes on the light cone, the polarization charges are given by the values of ϕ midway between the outgoing quarks. From Eq. (20) one sees that the polarization charges are equal and opposite to the charges of the outgoing quarks. The polarization charge can combine with the outgoing charge when the distance between the two becomes $\approx m^{-1}$ in the rest frame of the outgoing quark. This means a distance $\approx Q^{-1}$ in the c.m. frame of the original pair. This configuration occurs after a time $t\approx Q/m^2$. The proportionality between t and Q is important because it ensures that the outgoing fermions remain free for sufficiently long times to justify the calculations of the naive parton model.

The polarization current $j_\mu=\epsilon_{\mu\nu}\partial^\nu\phi$ and its lines of flow are along the hyperbolas of equal ϕ as in Fig. 11. It is clear from this that the mechanism which annihilates the outgoing fermions is an "inside-outside" cascade. In fact, the average flow of charge is very similar to the average flow of polarization charge shown in the diagram of Fig. 6.

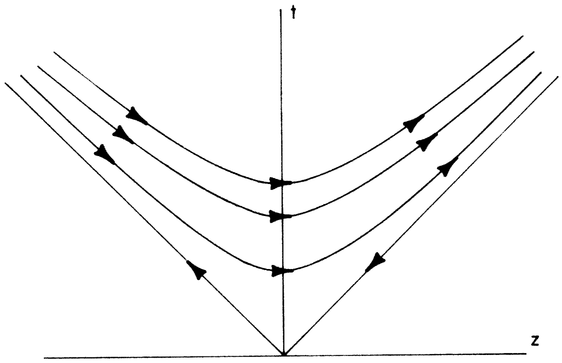

FIG. 11. Hyperbolas indicate lines of constant dipole density ϕ. The flow of current is indicated by the arrows.

The distribution of the emitted vector bosons can be calculated from the field ϕ. Since the field is coherent, the outgoing particles follow a Poisson distribution with a density on the rapidity axis of 1. To see this note that the quantity $\phi+\phi_{\text{ext}}$ is

$$\phi+\phi_{\text{ext}}=2g\int\frac{1}{p^2-m^2}\,e^{-ip\cdot x}\frac{d^2p}{(2\pi)^2}$$

$$=2g\int\frac{i}{2\omega_p}\,e^{i(p_1z-\omega_pt)}\frac{dp_1}{(2\pi)}\,, \tag{21}$$

where $\omega_p=(p^2+m^2)^{1/2}$. This expression should be compared with the second-quantized expression for the field [normalized by Eq. (9)]

$$m\int\frac{1}{(2\omega_p)^{1/2}}\,a_p^\dagger e^{ip\cdot x}\frac{dp_1}{(2\pi)^{1/2}}+\text{H.c.} \tag{22}$$

Thus, we identify the momentum-space number density

$$\langle a_p^\dagger a_p\rangle=\frac{1}{\omega_p}\,. \tag{23}$$

This is the familiar dx/x final-particle distribution which has been conjectured by Berman, Bjorken, and Kogut[7] and Feynman.[8]

It is also interesting to calculate the distance the initial fermions travel before losing a finite (small) fraction of their momenta. The momentum loss $\dot{P}$ is caused by the "electric" force[4] gE exerted by the polarization charge on the outgoing pair

$$\frac{dP}{dt}=gE. \tag{24}$$

In a world of one space and one time dimension, the "electric" field is related to the charge density via

$$\frac{\partial E}{\partial z}=\rho. \tag{25}$$

Outside the light cone the electric field is zero, so in the region between the light cone and the polarization charge there is a time-independent "electric" field of magnitude $\approx g$. Thus, the rate of momentum loss suffered by the outgoing fermions is independent of time and their momenta (until the polarization charge catches up). Therefore, the time it takes for the quark to lose a finite fraction of its momentum grows linearly with Q. Again, this suggests that the outgoing particles remain free for a sufficiently long time to justify the use of free-field theory in calculating short-distance and light-cone properties of the interacting theory. Since the analysis also shows that the outgoing quarks are always neutralized, this suggests that free-quark singularities will not appear in the matrix elements of products of currents. These points will be verified in the following discussions of the exact properties of quantum electrodynamics in one space and one time dimension.

IV. FREE FIELDS IN TWO DIMENSIONS

In this section we consider the properties of free Fermi fields in two dimensions. The fermi field can be expanded in terms of normal modes,

$$\psi(k)=\int\left[a(k)\psi(k)e^{-ik\cdot x}+b^\dagger(k)\psi(k)e^{ik\cdot x}\right]\frac{dk}{4\pi|k|}\,, \tag{26}$$

where a and $b^\dagger$ are destruction and creation operators for particles and antiparticles, respectively, normalized to

$$\{a(k),a^\dagger(k')\}=4\pi|k|\delta(k-k').$$

The fermions can be divided into four groups: right- or left-moving particles or anitparticles. Using the representation

$$\gamma_0=\begin{pmatrix}1&0\\0&-1\end{pmatrix},\quad\gamma_1=\begin{pmatrix}0&1\\-1&0\end{pmatrix}, \tag{27}$$

we obtain

$$\psi_R=\sqrt{k}\begin{pmatrix}1\\1\end{pmatrix},\quad\psi_L=\sqrt{k}\begin{pmatrix}1\\-1\end{pmatrix}, \tag{28}$$

where $\psi_{R,L}$ are spinors for right (left) movers.

We will consider two local currents $j^\mu=g\bar{\psi}\gamma^\mu\psi$, $s=\bar{\psi}\psi$. It is easy to see that the current $j^\mu(x)$ is the sum of two operators, one involving just right movers and the other just left movers.[9] Consider the quantity

$$T^{\mu\nu}_{\text{free}}(x)=-i\langle 0|Tj^\mu(x)j^\nu(0)|0\rangle\,. \tag{29}$$

The operator j^μ when acting on the vacuum can create two left movers or two right movers, but never a left mover and a right mover. Therefore, the intermediate spectrum of the correlation function Eq. (29) consists solely of lightlike momenta, so its only singularities in momentum space are at $p^2=0$. The function $T^{\mu\nu}_{\text{free}}(x)$ has the form

$$T^{\mu\nu}_{\text{free}}(x)=\frac{g^2}{\pi}(g^{\mu\nu}\Box-\partial^\mu\partial^\nu)\Delta_F(0,x^2). \tag{30}$$

Next consider the deep-inelastic structure function for absorption of a spacelike photon by a free fermion. Choose a frame in which the fermion's initial momentum is $(E,P_z)=(P,P)$ and the virtual photon's momentum is $(0,-2P)$. Since the electromagnetic current j^μ cannot convert a left mover into a right mover, the process cannot, in fact, occur and the structure function vanishes identically. The reason for this can be traced to the absence of transverse photons in 2 dimensions and the fact that spin-$\frac{1}{2}$ free quanta cannot absorb longitudinally polarized photons. The absence of an absorptive part for these two matrix elements is a special property of free fermions. For free bosons the corresponding structure function exists and is dimensionless. The annihilation process Eq. (29) has a constant, nonzero absorptive part in the boson case.

The operator $s(x)=\overline{\psi}(x)\psi(x)$ is more interesting because it connects left movers and right movers. It has nonzero structure functions and yields a constant absorptive part in the annihilation channel. This is clear by inspection of the matrix elements

$$S_{\text{free}}(q^2)=i\int e^{iq\cdot x}\langle 0|Ts(x)s(0)|0\rangle\,\frac{d^2x}{\pi} \tag{31}$$

and

$$W_{\text{free}}(q^2)=\operatorname{Im}S_{\text{free}}(q^2), \tag{32}$$

where $q^2>0$. Consider next the scattering channel. The structure function $W_{\text{free}}(q^2,q\cdot p)$ is given by

$$W_{\text{free}}(q^2,q\cdot p)=\operatorname{Im}\int ie^{iq\cdot x}\langle p|Ts(x)s(0)|p\rangle\,\frac{d^2x}{\pi}, \tag{33}$$

where $|p\rangle$ is a free fermion of momentum p. Equation (33) is easily evaluated,

$$\begin{aligned}W_{\text{free}}(q^2,q\cdot p)&=2p\cdot q\,\delta(2p\cdot q+q^2)\\&=\delta(\eta-1),\end{aligned} \tag{34}$$

where $\eta=-q^2/2q\cdot p$.

These scaling laws should be contrasted with those for bosons. Consider the vertex $s_b(x)=\phi^*(x)\phi(x)$ so that

$$W_b(q^2,q\cdot p)=\operatorname{Im}\int ie^{iq\cdot x}\langle p|Ts_b(x)s_b(0)|p\rangle\,\frac{d^2x}{\pi}. \tag{35}$$

For a free boson,

$$\begin{aligned}W_b&=\delta(2p\cdot q+q^2)\\&=\frac{1}{2p\cdot q}\delta(\eta-1).\end{aligned} \tag{36}$$

Thus, this scaling law carries different dimensions than Eq. (33). We will see later that although the spectrum of two-dimensional quantum electrodynamics consists only of bosons, the scaling laws for structure functions are characteristic of underlying Fermi fields.

V. GREEN'S FUNCTIONS IN TWO-DIMENSIONAL QUANTUM ELECTRODYNAMICS

We now turn to two-dimensional quantum electrodynamics. Consider first the single-fermion Green's function

$$G(x,0)=-i\langle 0|T\psi(x)\overline{\psi}(x)|0\rangle. \tag{37}$$

Since the theory is solved by

$$\begin{aligned}\psi(x)&=e^{-ig\alpha\Phi(x)}\chi(x),\\ \overline{\psi}(x)&=\overline{\chi}(x)e^{-ig\alpha\Phi(x)},\end{aligned} \tag{38}$$

where χ is a free Dirac field, G takes the form

$$G(x,0)=G_0(x)\langle 0|Te^{ig\alpha\Phi(x)}e^{-ig\alpha\Phi(0)}|0\rangle, \tag{39}$$

where G_0 is the free Dirac Green's function

$$G_0(x)=-(2\pi)^{-1}\frac{\gamma\cdot x}{x^2-i\epsilon}. \tag{40}$$

The second factor in Eq. (39) can be simplified to[10]

$$\exp\{-i\pi[\Delta_F(0,x^2)-\Delta_F(m^2,x^2)]\}, \tag{41}$$

where $\Delta_F(m^2,x^2)$ is the familiar Feynman propagator. Equation (39) becomes

$$G(x)=[G_0(x)e^{-i\pi\Delta_F(0,x^2)}]e^{i\pi\Delta_F(m^2,x^2)}, \tag{42}$$

where the factorization indicated on the right-hand side will be particularly illuminating. Since

$$i\pi\Delta_F(0,x^2)=-\tfrac{1}{4}\ln(x^2-i\epsilon)+\text{const} \tag{43}$$

we have

$$G(x^2)=-(2\pi)^{-1}\frac{\gamma\cdot x}{(x^2-i\epsilon)^{3/4}}e^{i\pi\Delta_F(m^2,x^2)}. \tag{44}$$

For large values of x^2 the factor $\exp[i\pi\Delta_F(m^2,x^2)]$ can be set equal to unity. From Eqs. (40) and (44) we see that $G(x^2)$ increases relative to $G_0(x^2)$ as a power $(x^2)^{+1/4}$. For small values of x^2, i.e., sufficiently near the light cone, $x^2<m^{-2}$, the logarithmic singularities of $\Delta_F(0,x^2)$ and $\Delta_F(m^2,x^2)$ cancel. Therefore,

$$G(x)\xrightarrow[x^2\to 0]{}G_0(x). \tag{45}$$

Thus, the short-distance and light-cone properties of the exact fermion propagator are characteristic of a free fermion.

For large times the exact propagator $G(x)$ tends to infinity relative to G_0. This has the odd consequence that the probability to find a fermion at a later time becomes infinite. This is, of course, a gauge-dependent interpretation and occurs because of the indefinite metric of quantum electrodynamics formulated in the Lorentz gauge. In the more physical Coulomb gauge the probability to find a single fermion at any later time is zero. (See Appendix A.) The physical reason for this is that in one dimension the electric field created by a single charge does not fall off at large distances. Therefore, the probability per unit time to create a fermion-antifermion pair from the vacuum is proportional to the volume of space. So, at any time after putting a fermion into the system there is vanishing probability to find only one fermion.

The peculiar character of the charged sectors of the theory can also be seen in the structure of the normal modes. As obtained in Appendix A, a right-moving normal mode χ_R is given

$$\chi_R(z,t)=\exp\left[ig\int_{-\infty}^{z}(\psi^\dagger\psi+\psi^\dagger\alpha\psi)\,dz'/m^2\right]\psi_R(z,t).$$

Therefore, the normal mode consists of a single fermion followed by an infinite line of polarized pairs. The actual charge of the system is not localized at position z. In fact, if the lower limit of the line integral appearing in the equation above was $-L$, we would find the charge located at $z=-L$. Thus the polarization charge is actually found at spatial infinity. For this reason the charged sectors of the theory can never be excited by the action of local sources.

Since no real asymptotic fermions exist in this theory, it follows that there are no fermion singularities in the matrix elements of the currents $j^\mu(x)$ and $s(x)$. Define[11]

$$T^{\mu\nu}(x)=-i\langle 0|Tj^\mu(x)j^\nu(0)|0\rangle. \qquad (46)$$

Using Eq. (7) gives

$$T^{\mu\nu}(x)=-i\epsilon^{\mu\sigma}\partial_\sigma\epsilon^{\nu\rho}\partial_\rho\langle 0|T\phi(x)\phi(0)|0\rangle$$
$$=\frac{g^2}{\pi}\,(g^{\mu\nu}\Box-\partial^\mu\partial^\nu)\Delta_F(m^2,x^2). \qquad (47)$$

Therefore, the intermediate spectrum is exhausted by one boson of mass $m=g/\sqrt{\pi}$, although the short-distance behavior is identical to that of the free Fermi field in two dimensions [Eq. (30)]. Consider the matrix element relevant to deep-inelastic scattering off a free boson of momentum p, i.e.,

$$\langle p|j^\mu(x)j^\nu(0)|p\rangle. \qquad (48)$$

Since $j^\mu(x)$ satisfies free-field equations of motion and commutation rules, only the disconnected piece of Eq. (48) is nonzero. Therefore, the associated structure function vanishes as in the free-field case.

Now consider a two-dimensional analog of $e^+e^-\to$ hadrons in which an external "scalar photon" couples into this theory via the operator $s(x)=\bar\psi(x)\psi(x)$. Recall that $s(x)$ generates an initial state of a quark and an antiquark which separate on opposite branches of the light cone. This is the state of interest in the real world. The relevant matrix element is

$$S(x)=i\langle 0|Ts(x)s(0)|0\rangle. \qquad (49)$$

The aim here will be to show that S is identical to the free-field $S_{\rm free}$ when $x^2\to 0$, but that no quarks appear in its intermediate states. Using the solution for ψ given in Eq. (15), Eq. (49) becomes

$$S(x)=i\langle 0|T\bar\chi(x)e^{-2ig\alpha\Phi(x)}\chi(x)\bar\chi(0)e^{-2ig\alpha\Phi(0)}\chi(0)|0\rangle. \qquad (50)$$

Equation (50) can be simplified

$$i\,\mathrm{tr}[G_0(x)]^2\langle 0|Te^{-2ig\alpha\Phi(x)}e^{-2ig\alpha\Phi(0)}|0\rangle$$
$$=\left[\frac{2i}{4\pi^2x^2}\,e^{-4i\pi\Delta_F(0,x^2)}\right]e^{4i\pi\Delta_F(m^2,x^2)} \qquad (51)$$

The short-distance behavior of $S(x)$ is evidently identical to the free fermion theory since as $x^2\to 0$, $\Delta_F(m^2,x^2)\to\Delta_F(0,x^2)$. To see how fermion singularities are replaced by boson singularities in the intermediate states of $S(x)$, recall Eq. (43). Thus, the free fermion factor x^{-2} is precisely cancelled by the first exponential in Eq. (51), leaving only an exponential of the massive boson propagator. This remaining exponential corresponds to a sum of Feynman diagrams in which any number of massive bosons are exchanged between points 0 and x (Fig. 12). Thus the singu-

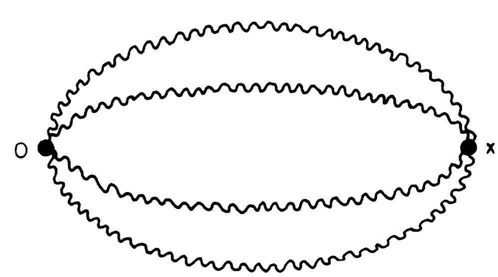

FIG. 12. Wavy lines indicate the exchange of any number of massive vector mesons between points 0 and x of a current correlation function.

larities of $S(x)$ consist of a series of thresholds at $m^2, 4m^2, \ldots$. In a similar manner it can be seen that matrix elements of products of currents $s(x)$ and $j^\mu(x)$ saturate with free massive bosons.[12]

Next consider the matrix element

$$T(x^2, x\cdot p) = i\langle \phi_{\text{in}}(p)| Ts(x)s(0)|\phi_{\text{in}}(p)\rangle_{\text{connected}} \tag{52}$$

whose absorptive part is

$$W(x^2, x\cdot p) = \langle \phi_{\text{in}}(p)| s(x)s(0)|\phi_{\text{in}}(p)\rangle, \tag{53}$$

where $|\phi_{\text{in}}(p)\rangle$ denotes a single-boson state of momentum p, normalized to

$$\langle p|p'\rangle = 4\pi E_p \delta(p-p').$$

Using the solution for ψ,

$$T(x^2, x\cdot p) = -16\pi \sin^2(p\cdot x/2) S(x^2), \tag{54}$$

where

$$S(x^2) = \frac{i}{2\pi^2(x^2 - i\epsilon)} \times \exp\{-4i\pi[\Delta_F(0,x^2) - \Delta_F(m^2,x^2)]\}. \tag{55}$$

The deep-inelastic structure function is defined by

$$W(q^2, q\cdot p) = \operatorname{Im}\int e^{iq\cdot x}T(x^2, x\cdot p)\frac{d^2x}{\pi} = \frac{1}{\pi}\operatorname{Im}T(q^2, q\cdot p) \tag{56}$$

as $q^2 \to -\infty$ with $\eta = -q^2/2q\cdot p$ held fixed. As is well known, this limit isolates the light cone $x^2=0$. But, as $x^2 \to 0$,

$$S(x^2) \underset{x^2\to 0}{\sim} \frac{i}{2\pi^2(x^2 - i\epsilon)} \tag{57}$$

since the Δ_F functions then cancel. Thus the scaling law for $W(q^2, q\cdot p)$ is identical to a theory with free fermions. $W(q^2, q\cdot p)$ can be calculated explicitly. Carrying out the Fourier transformation gives

$$\begin{aligned} T(q^2, q\cdot p) &= -16i\pi\int (x^2 - i\epsilon)^{-1}\exp\{-4i\pi[\Delta_F(0,x^2) - \Delta_F(m^2,x^2)]\}\sin^2(p\cdot x/2)e^{iq\cdot x}\frac{d^2x}{2\pi^2} \\ &= -8\pi S(q^2) + 4\pi[S(q^2(1-\omega)) + S(q^2(1+\omega))], \end{aligned} \tag{58}$$

where

$$S(q^2) = \int S(x^2) e^{iq\cdot x} d^2x, \tag{59}$$

and $\omega = -2p\cdot q/q^2$. The imaginary part of the second term gives the deep-inelastic structure function $e^- + \text{boson} \to e^- + \text{anything}$. From Eqs. (56), (57), and (58) we see that

$$W(q^2, q\cdot p) = W(s) \xrightarrow[q^2\to-\infty]{} W(\infty) = 2. \tag{60}$$

Thus, the deep-inelastic function scales in a particularly simple way.[13]

The function S also governs the single meson inclusive spectrum produced in the annihilation channel. The discontinuity of the first term on the right-hand side of Eq. (58) is proportional to the total cross section $e^+e^- \to$ anything. The last term is proportional to the inclusive spectrum $e^+e^- \to \text{boson} + \text{anything}$ in the physical region $q^2 > 0$, $0<\omega<1$. We find the height of the plateau

$$\frac{1}{\sigma}\frac{d\sigma}{dy} = 1, \quad y = \text{rapidity} \tag{61}$$

and, therefore, a total multiplicity of vector bosons,

$$\bar{n}_{\text{boson}}(Q^2) = \ln Q^2. \tag{62}$$

We note that this exact calculation agrees with the semiclassical calculations done earlier. The reason for this can be traced to the fact that the bosons are produced in a coherent state.

The mechanism causing the cancellation of the fermion singularities in matrix elements of currents is generally valid. A general matrix element of products $\bar\psi\psi$ will consist of loops of fermion propagators times exponentials of the form

$$\exp\{-4i\pi[\Delta_F(0,x^2) - \Delta_F(m^2,x^2)]\}.$$

The factors involving $\Delta_F(0,x^2)$ will cancel the singularities of the free fermion loops leaving only the singularities of the massive bosons. These are obtained by expanding the exponential factors involving $\Delta_F(m^2,x^2)$.

VI. ABELIAN TRIPLET MODEL

In this section we will illustrate a somewhat more physical model containing "mesons" and "baryons." Our purpose in constructing this model is to show the possibility of eliminating triality instead of fermion number by screening an additive quantum number similar to the charm of three-triplet models.[14] The "triplet" incorporated in this model represents the three "color" states of a quark and not the usual SU(3) label. The present model is based on an Abelian charm although we feel that the observed spectrum of hadrons requires a non-Abelian gauge field (cf. Sec. VIII). An interesting property of the model presented here is the absence of exotic hadrons in its spectrum, i.e., the asymptotic states are completely described in terms of $q\bar{q}$ and qqq bound states.

Consider a multiplet of fermions C_1, C_2, and C_3 carrying charm 2, −1, and −1, respectively. A vector gauge field $B_\mu(x)$ is coupled to the charm via the Lagrangian

$$\begin{aligned}\mathcal{L}_{\text{int}} &= g\bar{\psi}\gamma_\mu C\psi B^\mu \\ &= J_\mu^C B^\mu,\end{aligned} \tag{63}$$

where C is the matrix

$$C = \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{64}$$

In addition to the charm current J_μ^C, we define a baryon current

$$J_\mu^B = \tfrac{1}{3}\bar{\psi}\gamma_\mu\psi \tag{65}$$

and three additional boson currents J_μ^i,

$$J_\mu^i = \bar{\psi}\gamma_\mu T^i\psi, \tag{66}$$

where

$$T_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad T_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad T_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{67}$$

The technique of solution for this model is identical to the quantum-electrodynamics case. We obtain the following equation for the fermion field,

$$\chi(x) = e^{igC\alpha\Phi(x)}\psi(x), \tag{68}$$

where $\Phi = \pi(\tilde{\phi} - \phi)/g^2$. The scalar fields satisfy

$$\begin{aligned}&(\Box + m^2)\phi = 0, \quad \Box\tilde{\phi} = 0, \\ &[\phi(z,t), \dot{\phi}(z',t)] = im^2\delta(z-z'), \\ &[\tilde{\phi}(z,t), d\tilde{\phi}(z',t)/dt] = -im^2\delta(z-z'),\end{aligned} \tag{69}$$

where

$$\begin{aligned}m^2 &= (g^2/\pi)\operatorname{tr}C^2 \\ &= 6g^2/\pi.\end{aligned} \tag{70}$$

The currents J_μ^C, J_μ^B and J_μ^i satisfy

$$\begin{aligned}&\Box J^C = -6g^2/\pi J^C, \\ &\Box J^B = 0, \quad \Box J^i = 0,\end{aligned} \tag{71}$$

indicating the existence of bosons with mass-squared $6g^2/\pi$ and zero. In addition to these bosons the spectrum also contains massless three-quark states ("baryons"). Consider the quantity

$$\Psi_R = \psi_R^{C_1}\psi_R^{C_2}\psi_R^{C_3}. \tag{72}$$

The field Ψ_R is a composite field satisfying a massless free Dirac equation for a right mover,

$$\begin{aligned}\partial_+\Psi_R &= (\partial_+\psi_R^{C_1})\psi_R^{C_2}\psi_R^{C_3} + \cdots \\ &= gC_1 B_+\psi_R^{C_1}\psi_R^{C_2}\psi_R^{C_3} + \cdots \\ &= g(C_1 + C_2 + C_3)B_+\psi_R = 0.\end{aligned} \tag{73}$$

In obtaining the free-field equation for Ψ_R it was crucial that the sum of the charms of the constituent quarks is zero.

The model can also be solved in the Coulomb gauge. One finds that all states having nonzero charm do not propagate. Thus, the asymptotic states of the theory consist of charm-zero states. A simple counting argument shows that such states have the quantum numbers of any number of baryons and mesons. Here a baryon is defined as a bound state of fermions C_1, C_2, and C_3, and a meson consists of a C_i and a $\bar{C}_i$, a $\bar{C}_2$ and C_3, or a $\bar{C}_3$ and C_2. Exotic asymptotic states also appear, but are fully described in terms of noninteracting meson and baryon states.

VII. MODEL FOR POLARIZATION CURRENTS IN FOUR DIMENSIONS

It is obviously important to ask whether the effects we have described are special to two dimensions or whether they can be generalized to four dimensions. Schwinger argued that the vector mesons of a gauge theory in four dimensions can develop a mass if the coupling constant exceeds a certain critical value.[3] Let us assume that this will be the case. We can then address ourselves to the simpler space-time aspects of the mechanism. The massive vector particle in the theory should appear in the spectrum of the current and control its large-time development. Therefore, it is plausible that for times which are sufficiently large, the generalization of Eq. (5) to four dimensions reads

$$\begin{aligned}(\Box + m^2)j^\mu(x) &\equiv (\partial_t^2 - \partial_x^2 - \partial_y^2 - \partial_z^2 + m^2)j^\mu(x) \\ &= 0.\end{aligned}$$

This equation is, of course, not valid for short times. The vector potential A_μ satisfies

$$\Box A_\mu = j_\mu \tag{74}$$

in the Lorentz gauge, so again the difference $m^2 A + j$ satisfies a free massless Klein-Gordon equation. In the presence of an external current

$$\begin{aligned}&\Box A_\mu = j_\mu + j_\mu^{\text{ext}}, \\ &\Box j_\mu = -m^2(j_\mu + j_\mu^{\text{ext}}),\end{aligned} \tag{75}$$

so again $(m^2A_\mu+j_\mu)$ satisfies a free massless Klein-Gordon equation. It is convenient to introduce antisymmetric polarization tensors $P_{\mu\nu}$ and $P_{\mu\nu}^{\rm ext}$ such that

$$j^\mu=\partial_\nu P^{\mu\nu},\quad j_\mu^{\rm ext}=\partial^\nu P_{\mu\nu}^{\rm ext}. \tag{76}$$

One can solve Eq. (75) by finding a solution to

$$\Box P_{\mu\nu}+m^2P_{\mu\nu}=-m^2P_{\mu\nu}^{\rm ext}. \tag{77}$$

Let the external current again consist of two points of charge moving in opposite directions (along the z axis, say) at nearly the speed of light. Then $P_{\mu\nu}^{\rm ext}$ has only a (zt) component given by

$$P_{tz}^{\rm ext}=-g\theta(t+z)\theta(t-z)\delta(x)\delta(y). \tag{78}$$

Equation (77) is easily solved in momentum space. Define

$$P_{\mu\nu}(q)=\int e^{iq\cdot x}P_{\mu\nu}(x)\,d^4x. \tag{79}$$

Then Eq. (77) becomes

$$P_{tz}(q)=-\frac{2gm^2}{p_+p_-(p^2-m^2)}, \tag{80}$$

where $p_\pm=E\pm p_z$. In configuration space

$$P_{tz}(x)=-2g\int e^{ip_T\cdot x_T}[\Delta_R(m^2+p_T{}^2,t^2-z^2)-\Delta_R(0,t^2-z^2)]\,\frac{d^2p_T}{(2\pi)^2}, \tag{81}$$

where Δ_R is the retarded commutator in two dimensions. The polarization charge is obtained from Eq. (81) by differentiation with respect to z

$$j^0(x)=-2g\int e^{ip_T\cdot x_T}\left[\frac{\partial}{\partial z}\Delta_R(m^2+p_T{}^2,t^2-z^2)-\frac{\partial}{\partial z}\Delta_R(0,t^2-z^2)\right]\frac{d^2p_T}{(2\pi)^2}. \tag{82}$$

The total polarization charge on a transverse surface is

$$\int j^0(x)\,d^2x_T=-2g\frac{\partial}{\partial z}[\Delta_R(m^2,t^2-z^2)-\Delta_R(0,t^2-z^2)] \tag{83}$$

which is exactly the same result obtained for the two-dimensional model. The distance between the polarization charge and the outgoing fermion tends to zero as $m^{-2}t^{-1}$. Since the polarization charge must be confined to the interior of the light cone, the transverse spread of polarization charge is of order m^{-1} and does not increase with time. Thus, the polarization charge eventually forms a pancake which catches the outgoing fermion. The force which the pancake exerts on the outgoing fermion is the same as in the two-dimensional model since a pancake of charge is the source of a constant electric field.

The emitted particle spectrum can be calculated as in Sec. III. We obtain

$$\frac{dN}{dp_\parallel d^2p_T}\sim\frac{g^2}{p_\parallel(p_T{}^2+m^2)^2} \tag{84}$$

which shows the expected dx/x distribution together with a transverse momentum "cutoff." The multiplicity in e^+e^- annihilation is again expected to rise logarithmically with Q^2.

VIII. CONCLUDING REMARKS

This paper clearly leaves many questions unanswered. In closing we shall discuss several problems for the future.

Perhaps the most difficult and important point is to demonstrate the possibility of our mechanism operating in a four-dimensional quantum field theory. We require a vacuum so polarizable that any unscreened isolated "charge" cannot exist. This is equivalent to the disappearance of a massless gauge boson from the physical spectrum of the theory. To demonstrate this, consider Maxwell's equation

$$\nabla\cdot E=\rho. \tag{85}$$

Applying Gauss's theorem, we see that an isolated unscreened charge will produce a long-range (r^{-2}) field which can only occur if a massless particle is present in the spectrum. Conversely, any mechanism which totally screens the charge eliminates the massless particle from the theory.

A second necessary ingredient in the models we have considered is that an appropriate multiplicative quantum number be screened (set equal to one, say, in the physical sector of the theory) when an additive quantum number is totally screened. For example, in the two-dimensional quantum electrodynamics model the fermion number f is defined to be -1 for fermions and $+1$ for bosons. We observe that

$$\begin{aligned}f&=(-1)^{N_f+N_{\bar f}}\\&=(-1)^{N_f-N_{\bar f}},\end{aligned} \tag{86}$$

where $N_{f(\bar f)}$ is the total number of fermions (antifermions) present. Since the total charge Q of the system is proportional to $N_f-N_{\bar f}$, requiring that Q be zero implies that $f=+1$. A similar connection between triality and charm can be demonstrated in the model of Sec. VI.

The phenomenon of total screening is *not* special to one-dimensional physics. Many examples of total screening can be cited: The most familiar illustration is the occurrence of complete screening in metals. When an isolated charge is introduced into a conductor, polarization charge immediately flows towards it from the surface and completely neutralizes it. In fact, if a positive and negative charge which recede from one another at less than the speed of sound are introduced into the metal, current then flows between them and eventually neutralizes them.

Another simple example of complete screening in four dimensions is the Abelian Higgs-Kibble-Guralnik-Hagen-Englert-Brout mechanism.[15] In this case the vector gauge field acquires a mass; however, the fermions are not removed from the spectrum of states. This follows from the fact that these theories have an additional charged scalar field (Higgs boson). Thus one cannot identify the total charge with the fermion number and therefore the screening of the additive quantum number ("charge") does not imply the complete screening of the multiplicative quantum number ("fermion number"). In these theories when a charged fermion is placed in the vacuum, it is screened by the charged boson field. Our mechanism requires that the algebraic structure of the theory be such that setting an additive quantum number to zero implies that the appropriate multiplicative quantum number (triality) be completely screened. Schwinger has speculated that complete screening can take place in four dimensions without Higgs' bosons if the coupling constant of the theory exceeds a certain critical value.[3]

Another topic to study is the precise algebraic properties of a gauge theory which realizes the mechanism we have suggested. Three-triplet models appear to provide the most natural framework for accomplishing this, because they possess an additive conserved quantum number whose vanishing implies the lack of triality. For example, an SU(3) degree of freedom could be superimposed on top of the charm degree of freedom in the model discussed in Sec. VI. A more aesthetic possibility is to couple the eight SU(3)′ (charm) currents to non-Abelian gauge fields and demand complete screening of all eight currents. This scheme would then require that the physical spectrum consist only of SU(3)′ singlets. Such models are presently empirically favored by known features of the hadron spectrum. Two possibilities which immediately come to mind are the Han-Nambu and Gell-Mann three-triplet models.[14] In the Han-Nambu model

$$Q = I_3 + \tfrac{1}{2}(Y + Y'),$$

where Y' is the hyperchargelike generator of SU(3)′, while

$$Q = I_3 + \tfrac{1}{2}Y$$

in Gell-Mann's version of "colored" quarks. Our mechanism can apply in the Gell-Mann colored-quark model as it stands. However, it cannot operate in the simplest version of the Han-Nambu model. This is so because the electromagnetic current is not a charmed singlet in this case, so it can create a state with nonzero total charm. Such a state would be highly unstable and would possess similar peculiarities as the charged sector of one dimensional quantum electrodynamics. A possible alternative is to introduce another SU(3)″ degree of freedom into the Han-Nambu model, i.e., three-nonets of Han-Nambu quarks. Then non-Abelian gauge fields would couple to the SU(3)″ currents and cause the physical sector of the theory to be a SU(3)″ singlet. The advantage of this scheme is that it allows the quarks to carry integral electric charge. In such a theory charmed [SU(3)′] hadrons would be produced at high enough energies, but triality-carrying states would always be forbidden. For example, in this model the electron-positron annihilation cross section into hadrons would be twelve times the electron-positron → muon-antimuon cross section.

Another question we have not addressed is whether Bjorken scaling can occur in four-dimensional gauge theories. We have instead argued that the mechanism which neutralizes the quark charges does not modify the short-distance behavior of the field theory. Recent studies of non-Abelian gauge theories have shown that they can be free at short distances.[16] However, it is not yet known whether these theories can explain the early-scaling behavior observed in the SLAC deep-inelastic experiments.

The best experimental check of our ideas involves the detection of final-state jets of hadrons produced in deep-inelastic reactions.[17] The momentum-space distribution of hadrons in a jet should be given by $dp_{\parallel}/E$ ($p_{\parallel}$ measured along the direction of the struck parton). In particular, there should be no large gaps in momentum space between the fastest hadrons in a jet and the fragments of the target. In fact, the picture of final hadron distributions should be essentially the same as that discussed by Berman, Bjorken, and Kogut,[7] and Feynman.[8]

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APPENDIX A: COULOMB-GAUGE SOLUTION OF TWO-DIMENSIONAL QUANTUM ELECTRODYNAMICS

The phenomena discussed in this paper are best visualized in the Coulomb gauge.[18] In this gauge there are no unphysical negative-metric particles to obscure the propagation properties of the fermion fields. The Coulomb gauge is defined by

$$A_1 = 0. \tag{A1}$$

The Dirac equation becomes

$$i(\gamma_0\partial_0 + \gamma_1\partial_1)\psi = g\gamma_0 A_0\psi. \tag{A2}$$

We will consider the propagation of a right-moving fermion defined by

$$\psi_R = \alpha\psi_R, \tag{A3}$$

where $\alpha = \gamma_0\gamma_1$. Now Eq. (A2) becomes

$$i\partial_+\psi_R = gA_0\psi_R, \tag{A4}$$

where

$$\partial_\pm = \frac{\partial}{\partial t} \pm \frac{\partial}{\partial z}.$$

Define a scalar field $\hat{\phi}$ by

$$A_0 = \partial_+\hat{\phi}. \tag{A5}$$

The solution to the Dirac equation then reads

$$\psi_R = \exp(-ig\hat{\phi})\chi_R, \tag{A6}$$

where χ_R satisfies a free, massless Dirac equation

$$i\partial_+\chi_R = 0.$$

Maxwell's equations in the Coulomb gauge imply

$$-\partial_1{}^2 A_0 = g\psi^\dagger\psi$$

and (A7)

$$\partial_0\partial_1 A_0 = g\psi^\dagger\alpha\psi.$$

It is convenient to define

$$J_\pm = g\psi^\dagger(1 \pm \alpha)\psi. \tag{A8}$$

Then Eqs. (A7) and (A8) give

$$-\partial_1\partial_+ A_0 = J_-. \tag{A9}$$

In terms of $\hat{\phi}$ this reads

$$\hat{\phi} = -\partial_+{}^{-2}\partial_1{}^{-1}J_-. \tag{A10}$$

From Eq. (5) in the text we recall that J_μ satisfies a massive Klein-Gordon equation

$$\partial_+\partial_- J_- = -m^2 J_-, \quad m^2 = g^2/\pi$$

or

$$\partial_+{}^{-1}J_- = -\partial_- J_-/m^2.$$

Substituting into Eq. (A10) and using current conservation, $\partial_+ J_+ + \partial_- J_- = 0$, gives

$$\hat{\phi}(z,t) = -\partial_1{}^{-1}J_+/m^2$$
$$= -\int_{-\infty}^{z} J_+(z',t)\,dz'/m^2. \tag{A11}$$

Therefore, the normal mode χ_R of the fermion field may be expressed in terms of the local field ψ_R by

$$\chi_R(z,t) = \exp\left(-i\pi\int_{-\infty}^{z}\psi^\dagger(1+\alpha)\psi\,dz'\right)\psi_R(z,t). \tag{A12}$$

The fermion Green's function can be calculated in this gauge and interpreted physically. Write J_μ in terms of a scalar field ϕ as in the text

$$J_\mu = \epsilon_{\mu\nu}\partial^\nu\phi,$$

where ϕ is a massive free field. For $t>0$, the Green's function for a right mover reads

$$\langle 0|\psi_R^\dagger(z,t)\psi_R(0,0)|0\rangle = \langle 0|\chi_R^\dagger(z,t)\chi_R(0,0)|0\rangle$$
$$\times\left\langle 0\left|\exp\left\{i\frac{\pi}{g}\int_{-\infty}^{z}\left[\dot{\phi}(z',t) - \frac{\partial\phi(z',t)}{\partial z'}\right]dz'\right\}\exp\left\{i\frac{\pi}{g}\int_{-\infty}^{0}\left[\dot{\phi}(z',0) - \frac{\partial\phi(z',0)}{\partial z'}\right]dz'\right\}\right|0\right\rangle. \tag{A13}$$

Using the normal mode expansion for the massive scalar field

$$\phi(z,t) = \frac{im}{2\sqrt{\pi}}\int[a^\dagger(k)e^{ik\cdot z} - a(k)e^{-ik\cdot z}]\frac{dk}{\sqrt{\omega_k}}, \tag{A14}$$

where

$$[a^\dagger(k), a(k')] = \delta(k-k'),$$

we obtain

$$\langle 0|\psi_R^\dagger(z,t)\psi_R(0,0)|0\rangle$$
$$= \langle 0|\chi_R^\dagger(z,t)\chi_R(0,0)|0\rangle e^{K(z,t)}, \tag{A15}$$

where

$$K(z,t)=\text{const}\times\int\left(e^{i(k\cdot z-\omega_k t)}-1\right)\frac{(\omega_k-k)^2}{k^2\omega_k}\,dk. \tag{A16}$$

For $t=0$, the integral defining $K(z,t)$ converges and tends to zero as $z\to 0$. Thus, for $t=0$ and small z, the propagator Eq. (A15) is identical to the free-field propagator. However, for $t\neq 0$, the integral diverges for all values of z and the propagator vanishes. The physical reasons for this behavior are discussed in the text.

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[1]J. D. Bjorken and E. A. Paschos, Phys. Rev. 185, 1975 (1969).

[2]In fact, this hypothesis might be overly conservative. For example, nonrelativistic quark models suppose that quarks are bound in extremely deep scalar "potentials." The meaning of these ideas and their relation to parton models formulated in an infinite-momentum frame and field theory has not been worked out.

[3]J. Schwinger, Phys. Rev. 128, 2425 (1962); in *Theoretical Physics*, Trieste Lectures, 1962 (I.A.E.A., Vienna, 1963), p. 89.

[4]"Electric" field refers here to the components of a strong-interaction gauge field.

[5]J. D. Bjorken, private communication to one of us (J.K.). Also, J. D. Bjorken, informal remarks during the Deep Inelastic Phenomenology session of the New York Academy of Science Conference, Recent Advances in High Energy Physics, 1973, Barbizon Hotel, New York City (unpublished).

[6]J. Kogut, D. K. Sinclair, and Leonard Susskind, Phys. Rev. D 7, 3637 (1973); 8, 2746(E) (1973).

[7]S. M. Berman, J. D. Bjorken, and J. B. Kogut, Phys. Rev. D 4, 3388 (1971).

[8]R. P. Feynman, *Photon Hadron Interactions* (Benjamin, New York, 1972).

[9]The simplicity of the quantum-number structure of this model prevents us from constructing an electromagnetic current which is distinct from the screened strong interaction current. In the model of Sec. VI there is additional structure and no such ambiguity.

[10]A careful calculation retaining all multiplicative factors replaces $\Delta_F(m^2,x^2)\to\Delta_F(m^2,x^2)-\Delta_F(m^2,0)$ and $\Delta_F(0,x^2)\to\Delta_F(0,x^2)-\Delta_F(0,0)$. With this replacement the matrix elements are both infrared- and ultraviolet-convergent.

[11]Because of the presence of Schwinger terms, the time-ordered product should actually be replaced by a T^* operation in order that $T^{\mu\nu}(x)$ be covariant. A consistent treatment of these subtleties agree with the naive manipulations done here.

[12]It should be noted that the matrix elements of $\bar\psi\psi$ as calculated here do not have good cluster properties. In particular the matrix element of Eq. (49) does not tend to zero as $x\to\infty$ although $\langle 0|\bar\psi\psi|0\rangle=0$. The bad cluster properties can be removed by a more suitable choice of the vacuum state as discussed by J. Lowenstein and J. Swieca, Ann. Phys. (N.Y.) 68, 172 (1971). The ambiguity in the choice of vacuum is due to the spontaneous breakdown of chiral symmetry. With the better choice of vacuum the right-hand side of Eq. (51) is replaced by $\cos 4\pi\Delta_F(m^2,x^2)$. One of the authors (L.S.) thanks C. Callan and S. Coleman for a helpful discussion of these points.

[13]A parton sum rule expressing the conservation of total momentum can be derived assuming that the massive bosons are a collection of fermions. It reads $\int_0^1 W(q^2,q\cdot p)d\eta=1$. This sum rule is violated by a factor of two in the present model. This is perhaps not too surprising because $s(x)=\bar\psi(x)\psi(x)$ is not a good component of a conserved current.

[14]The first three-triplet model was proposed by M. Y. Han and Y. Nambu, Phys. Rev. 139, B1006 (1965). In this case "charm" can couple directly with external probes and the quarks have integer charges. Other models are described by H. Fritzsch and M. Gell-Mann, in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 2, p. 132.

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